STA 131 B: Mathematical Statistics **Spring 2024**

Homework 2

Lecturer: Hang Zhou *Due date: 2:00pm, April 18*

Homework is a crucial step in your learning journey for this course, enriching your understanding of mathematical statistics. I strongly suggest you spend time on it and complete it independently.

Question 1: Assume $X_i \sim_{i.i.d.}$ Unif$([\theta, \theta+1])$, $\Theta \in \mathbb{R}$. Find the MLE of θ.

Question 2: Assume $X_i \sim_{i.i.d.}$ Exponential(λ), that is, the exponential distribution with parameter λ, with pdf

$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}.$$

The parameter space $\Theta = (0, \infty)$. Assume at least one X_i is non-zero, find the MLE of λ.

Question 3: Assume $X_i \sim_{i.i.d}$ Unif$([\alpha, \beta])$. The pdf for X_i is

$$f(x|\alpha, \beta) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases}.$$

a). Write down a reasonable parameter space for $\theta = (\alpha, \beta)$.

b). Find the methods of moments estimator for α and β.

[hints: you may need the equality $b^2 - a^2 = (b-a)(b+a)$ and $b^3 - a^3 = (b-a)(a^2 + ab + b^2)$]

Question 4: Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with density function

$$f(x|\sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right), \ \theta \in \Theta = (0, \infty).$$

a). Assume at least one X_i is none-zero, find the maximum likelihood estimate of σ.

b). Find the methods of moments estimator of σ.

[hints: 1. if f is an odd function, that is $f(x) = -f(-x)$, then $\int_{-a}^{a} f(x)dx = 0$ for all $a > 0$. 2. you can use *Integration by parts* `https://en.wikipedia.org/wiki/Integration_by_parts` to calculate the integral $\int y^2 e^{-y} dy$ and $\int y e^{-y} dy$.]