| STA 131 B: Mathematical Statistics | Spring 2024 |
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# Homework 3

*Lecturer: Hang Zhou* — *Due date: 2:00pm, May 2*

*Homework is a crucial step in your learning journey for this course, enriching your understanding of mathematical statistics. I strongly suggest you spend time on it and complete it independently.*

**Question 1:** Assume $X_1, \ldots, X_n \sim_{i.i.d.} \mathcal{N}(\mu, \sigma_0^2)$ with *unknown* mean $\mu$ and *known* variance $\sigma_0^2$. Assume the prior distribution for $\mu$ is $\mu \sim \mathcal{N}(\eta, \delta^2)$. Find the posterior distribution of $\mu$.

**Question 2:** Assume $X_i \sim_{i.i.d.}$ Exponential($\lambda$), that is, the exponential distribution with parameter $\lambda$, with pdf

$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}.$$

Assume the prior distribution for $\lambda$ is $\lambda \sim \Gamma(\alpha, \beta)$.

a) Find the posterior distribution of $\lambda$.

b) Find the Bayes estimator of $\lambda$.

hint: if $X \sim \Gamma(\alpha, \beta)$, then the pdf of $X$ is

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}.$$

$\mathbb{E}(X) = \alpha\beta^{-1}$, $\mathbb{E}(X^2) = \alpha(\alpha+1)\beta^{-2}$.

**Question 3:** Assume $X_i \sim_{i.i.d.}$ Poisson($\theta$), that is, the Poisson distribution with parameter $\theta$, with pdf

$$f(x|\theta) = \frac{e^{-\theta}\theta^{X_i}}{X_i!}.$$

Assume the prior distribution for $\theta$ is $\theta \sim \Gamma(\alpha, \beta)$.

a) Find the posterior distribution of $\theta$.

b) Find the Bayes estimator of $\theta$.

**Question 4:** Assume $X_i \sim_{i.i.d.}$ Unif($[0, \theta]$). Assume the prior distribution for $\theta$ is $\theta \sim$ Pareto($x_0, \alpha$) with pdf

$$f(\theta \mid x_0, \alpha) = \frac{\alpha x_0^\alpha}{\theta^{\alpha+1}} \mathbb{1}_{\{\theta \geq x_0\}}, x_0 > 0.$$

Find the posterior distribution of $\theta$.