STA 131 B: Mathematical Statistics	**Spring 2024**

Homework 4

Lecturer: Hang Zhou | *Due date: 2:00pm, May 9*

Homework is a crucial step in your learning journey for this course, enriching your understanding of mathematical statistics. I strongly suggest you spend time on it and complete it independently.

Question 1: Suppose we have random samples $X_1, \ldots, X_n$ from a $\mathcal{N}\left(\theta, \sigma^2\right)$ distribution where σ^2 is known and is therefore not a parameter of interest. Show that $T = \sum_{i=1}^n X_i$ is the sufficient statistic for θ.

Question 2: Let $X_1, \ldots, X_n \sim_{i.i.d.} \mathcal{N}\left(\mu, \sigma^2\right)$ where μ is known and σ^2 is unknown. Show that $T = \sum_{i=1}^n \left(X_i - \mu\right)^2$ is sufficient for σ^2.

Question 3: Let $X_1, \ldots, X_n$ are random samples from geometric distribution with pmf

$$\mathbb{P}(X = x) = \theta(1-\theta)^x,\ x = 0, 1, \ldots,$$

where $\theta \in (0, 1)$ is the unknown parameter.

a) Use Factorization theorem to show that $T = \sum_{i=1}^n X_i$ is the sufficient statistic for θ.

b) Use definition to show that $T = \sum_{i=1}^n X_i$ is the sufficient statistic for θ, that is, you need to show there is no θ in the conditional distribution of $(X_1, \ldots, X_n)|T = t$.

[hint]: In this example, the statistic $T = \sum_{i=1}^n X_i$ has the Pascal distribution with pmf

$$\mathbb{P}(T = t|\theta) = \binom{t+n-1}{n-1}\theta^n(1-\theta)^t, \quad t = 0, 1, \cdots.$$

[hint]: You may use the equality

$$\mathbb{P}\left(X_1 = x_1, \cdots, X_n = x_n, T = t\right) = \mathbb{P}\left(X_1 = x_1\right) \cdots P\left(X_{n-1} = x_{n-1}\right) \mathbb{P}\left(X_n = t - \sum_{i=1}^{n-1} x_i\right).$$

Question 4: Let $X_1, \ldots, X_n$ are random samples from Poisson distribution with pmf

$$\mathbb{P}(X = k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!},\ k = 0, 1, \ldots,$$

where $\lambda \in (0, \infty)$ is the unknown parameter.

a) Use Factorization theorem to show that $T = \sum_{i=1}^n X_i$ is the sufficient statistic for λ.

b) Use definition to show that $T = \sum_{i=1}^n X_i$ is the sufficient statistic for λ, that is, you need to show there is no λ in the conditional distribution of $(X_1, \ldots, X_n)|T = t$.

[hint]: In this example, the statistic $T = \sum_{i=1}^n X_i$ has the Poisson distribution with parameter $n\lambda$, that is, $T \sim \text{Poisson}(n\lambda)$.