STA 131 B: Mathematical Statistics **Spring 2024**

Homework 6

Lecturer: Hang Zhou *Due date: 2:00pm, May 30*

Homework is a crucial step in your learning journey for this course, enriching your understanding of mathematical statistics. I strongly suggest you spend time on it and complete it independently.

Question 1: Assume $X_1 \dots, X_n$ are random samples from the distribution with pdf

$$f(x|\theta) = \theta^2 x e^{-\theta x} \mathbb{1}_{\{x>0\}}.$$

Find the UMVUE of θ^{-1}.
[*hint*]:

- The probability density function for the Gamma distribution $X \sim \text{Gamma}(\alpha, \beta)$ is

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \text{ for } x > 0.$$

- If $X \sim \text{Gamma}(\alpha, \beta)$, $\mathbb{E}(X) = \alpha/\beta$.
- $T = \sum_{i=1}^n X_i \sim \text{Gamma}(2n, \theta)$.

Question 2: Assume $X_1 \dots, X_n$ are random samples from Poisson(λ) with pmf

$$f(x|\theta) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, ...$$

Find the UMVUE of λ. [*hint*]:

- If $X \sim \text{Poisson}(\lambda)$, $\mathbb{E}(X) = \lambda$.
- $T = \sum_{i=1}^n X_i \sim \text{Poisson}(n\lambda)$.

Question 3: Assume $X_1 \dots, X_n$ are random samples from Poisson(λ) with pmf

$$f(x|\theta) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, ...$$

Find the CR lower bound of unbiased estimators of λ.

Question 4: Assume $X_1 \dots, X_n$ are random samples from Exponential(λ) with pdf

$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0. \end{cases}$$

Find the CR lower bound of unbiased estimators of $1/\lambda$.